The Role of Topography in Geodetic Gravity Field Modelling

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Abstract: Masses associated with the topography, bathymetry, and its isostatic compensation are a dominant source of gravity field variations, especially at shorter wavelengths. On global scales the topographic/isostatic effects are also significant, except for the lowest harmonics. In practice, though, global effects need not be taken into account as such effects are included in the coefficients of the geopotential reference fields. On local scales, the short-wavelength gravity variations due to the topography may, in rugged terrain, be an order of magnitude larger than other effects. In such cases, explicit or implicit terrain reduction procedures are mandatory in order to obtain good prediction results. Such effects may be computed by space-domain integration or by FFT methods. Numerical examples are given in the paper for areas of the Canadian Rockies.

In principle, good knowledge of the topographic densities is required to produce the smoothest residual field. Densities may be determined from sample measurements or by gravimetric means, but both are somewhat troublesome methods in practice. The use of a standard density, e.g., 2.67 g/cm³, may often yield satisfactory results and may be put within a consistent theoretical framework.

The independence of density assumptions is the key point of the classical Molodensky approach to the geodetic boundary value problem. The Molodensky solutions take into account that land gravity field observations are done on a non-level surface. Molodensky's problem may be solved by integral expansions or more effective FFT methods, but the solution should not be intermixed with the use of terrain reductions. The methods are actually complimentary and may both be required in order to obtain the smoothest possible signal, least prone to aliasing and other effects coming from sparse data coverage, typical of rugged topography.

Introduction

The two aspects of the role of the topography, namely the direct attraction of masses of the terrain and the uneven surface on which terrestrial measurements are made, are from a theoretical point of view completely different problems. The first assumes a density model, while the second – the Molodensky theory – in principle is free of any density assumptions. Using terrain reductions a computational smoothing of the gravity field is attempted, making interpolation and prediction from scattered data points more precise. Molodensky's theory makes the classical geodetic boundary value problem solutions "correct" on the uneven topographic surface; applying the Molodensky correction terms to gravity implies no smoothing at al.. The methods are therefore complimentary and should be used together whenever feasible.

Terrain reductions

The terrain reductions may be classified under global or local models. On global scales, topographic-isostatic reductions must be used according to some idealized isostatic models. The simple model coming closest to geophysical reality is the Vening Meinesz model, which is a modified Airy model taking into account the elasticity of the crust, permitting short-wavelength loads to remain uncompensated. Some areas of the earth are, however, notably deviating from the simple models, as for example trench areas and midoceanic ridges. For a review

of computation of global topographic-isostatic effects, which may be done efficiently by fast spherical harmonic expansions, see Rummel et al. (1988).

The global topographic-isostatic reductions have only limited use in geodetic gravity field modelling, but are very useful for identifying anomalously compensated areas. The real impact of terrain reductions come on the local scale, where they strongly diminish aliasing from undersampling of the rapidly varying, height-correlated gravity anomalies in rugged topography.

On a local scale, isostatic effects need not be considered. Instead, a residual terrain model reduction (RTM), where only the topographic deviations from a smooth mean topography are considered, may be used with advantage. The smooth mean height surface h_r may be obtained from the topographic heights h by a moving average filter over suitable caps, say 1° in size. In this case, RTM-reduced gravity anomalies will resemble isostatic anomalies (Forsberg, 1984). Removing the complete effect of topography relative to a constant mean elevation level for a given area, as often done in practice, may be considered a special case of the RTM reduction. The total computational removal of all topography down to the geoid, i.e. the complete Bouguer reduction, is not useful in geodetic gravity field modelling because of very large indirect effects on the geoid.

The general form of a terrain effect on any gravimetric quantity expressable as a linear functional L(T) of the anomalous potential T is of the general form

$$L_p(T_m) = G \iint_E \int_{h_r}^{h} L_p(\frac{1}{\ell}) dx dy dz, \quad \ell = \sqrt{(x-x_p)^2+(y-y_p)^2+(z-h_p)^2} \quad (1)$$

where E is the infinite x-y plane (planar approximation). Integrals like (1) may in practice be evaluated by prism integration or by expansions in convolutions, permitting use of FFT techniques. For details see Forsberg (1984, 1985) or Sideris (1985). Formally T_m is the potential generated by the selected terrain mass model. When computations are done consistently in a remove-restore technique, i.e. modelling the reduced potential

$$T^c = T - T_m \quad (2)$$

by subtracting terrain effects from input data, and restoring terrain effects in predictions, then in principle density ρ need not be known. However, meaningful results are only obtained when ρ is close to the real word values.

The question of density

Estimation of good insitu densities is often quite difficult. Only the surface of topography is available for sampling, and measurements of bulk densities on rock samples tend to show high variability even within the same geological formation. For sedimentary rocks, questions of porosity, water saturation, and commpaction present special problems. Typical density values encountered in practice range from below 2.0 g/cm³ in moraine hills up to 3.3 g/cm³ in (rare) gabbroic intrusive areas. However, the standard density 2.67 g/cm³ represents a surprising good value in many cases (granities, gneisses, old sediments), and its use have been justified by many empirical investigations (Dobrin, 1976). Where significantly lower density values need to be used (e.g., young sediments), topographic relief is usually also lower and good density values are therefore less critical.

For local applications a good alternative to density measurements is the estimation of ρ through studies of correlation of gravity with topographic heights ("the Nettleton method"). This method may be put within a consistent framework in least-squares collocation, estimating one or more density parameters alongside the gravity field modelling itself. For details, see Sünkel (1981).

Molodensky's problem

The first order Molodensky solution to the geodetic boundary value problem consists of a series expansion of the form

$$\zeta(P) = \sum_{n=0}^{\infty} \frac{1}{2\pi\gamma} \iint_E \frac{g_n(Q)}{[(x_Q-x_P)^2+(y_Q-y_P)^2]^{\frac{1}{2}}} \, dx_Q dy_Q \qquad (3)$$

with

$$g_0(Q) = \Delta g(Q) \qquad (4)$$

$$g_1(Q) = -(h_Q-h_P) \frac{1}{2\pi} \iint_E \frac{\Delta g(x,y) - \Delta g(x_Q,y_Q)}{[(x_Q-x)^2+(y_Q-y)^2]^{3/2}} \, dxdy \qquad (5)$$

For a review, see Moritz (1980) or Sideris (1987). Depending on the terrain roughness, higher-order terms may be quite significant. Their computation requires repeated applications of the harmonic continuation integral (5), which may be formulated as a sequence of convolutions and evaluated efficiently by FFT methods. For examples see Sideris (1987).

When used on free-air anomalies, formula (5) is known to be closely related to the classical terrain correction c (Moritz, 1980). The relationship comes from noting that in rugged topography free-air anomalies show a correlation with terrain height of the form $\Delta g = \Delta g_B + 2\pi G\rho h$ where Δg_B is the simple Bouguer anomaly. Unfortunately the use of c (requiring density information) rather than g_1 (independent of ρ) have been the source of much confusion, and the practice is not recommended. It is thus preferable to evaluate (5) with terrain-reduced gravity data Δg^c. The corresponding "reduced" Molodensky terms g_n^c will be smaller, and the convergence of (3) improved. If terrain reductions are not used then, on the one hand, gravity data must be given densely enough to sufficiently sample even the shortest topographic-induced wavelengths, and, on the other hand, higher-order Molodensky series terms should be considered. Dense gravity data are hardly ever available in practice.

An example: Kananaskis area, Canadian Rocky Mountains

The Kananaskis area west of Calgary is a mountainous area with topography ranging from 1400 m to 3400 m. A number of astronomic deflections of the vertical and GPS-derived geoid undulations are available along the main valley area, in addition to gravity data space every 5 to 10 km in the surrounding region, and a dense digital terrain model. Results for a FFT gravity field modelling example are given below; for more results on terrain corrections for gravity and gradiometry in the same area, see Tziavos et al. (1988).

For the terrain reduction of the available data, a 100 m x 100 m and 1 km x 1 km DTM was used. A smooth height surface with resolution of 70 km was generated by averaging. The statistics of the data, removing topographic effects relative to the mean height surface, is shown below. Predictions were not attempted without terrain reductions, as individual gravity anomaly values could change up to 100 mgal close to the prediction points, depending on whether the observation happened to be made on top of a mountain or at the bottom of a valley. In other words, the observed Δg-field is seriously undersampled without some kind of terrain reduction.

Data type	Δg (mgal)		ξ (arcsec)		η (arcsec)		ζ(m, relative to northernmost GPS-point)	
No. of points	473		15				10	
Statistics	mean	std.dev.	mean	std.dev.	mean	std.dev.	mean	std.dev.
Observed values	20.5	57.2	1.37	3.37	.95	6.28	1.15	.46
Terrain effects	-14.9	47.2	.86	2.97	-.22	5.74	.27	.21
Reduced data	35.4	29.8	.51	1.60	1.16	2.15	.88	.25

For the FFT prediction, gravity anomalies were gridded on a 1.5'x2.5' grid in a $2.5° \times 3°$ region. A similar 1.5'x2.5' height grid was obtained by gridding gravity station heights for representing the uneven surface to which observations refer, in order to be able to use FFT techniques for computing the first-order Molodensky corrections g_1^c. The procedure of gridding station heights rather than averaging the detailed DTM is preferable, because the height distribution of gravity stations does not necessarily follow the averaged topography; gravity stations tend to be located in valleys rather than mountaintops.

The results of predictions with and without a spherical harmonic reference field (OSU86F to degree 180), and with and without the Molodensky terms, are shown below. Considering the rough topography, results are very satisfactory. The influence of the Molodensky term seems to be completely masked by other error sources, illustrating the high degree of smoothness locally provided by the terrain reductions.

Kananaskis: Prediction results (observed minus predicted) for various solution types

Solution with		ξ (arcsec)		η (arcsec)		ζ relative (m)	
180x180 reference field	Molodensky g_1 terms	mean	std.dev.	mean	std.dev.	mean	RMS
No	No	.92	.65	.31	.57	.09	.24
No	Yes	.91	.69	.33	.60	.09	.25
Yes	No	1.92	.66	-1.57	.48	.09	.17

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